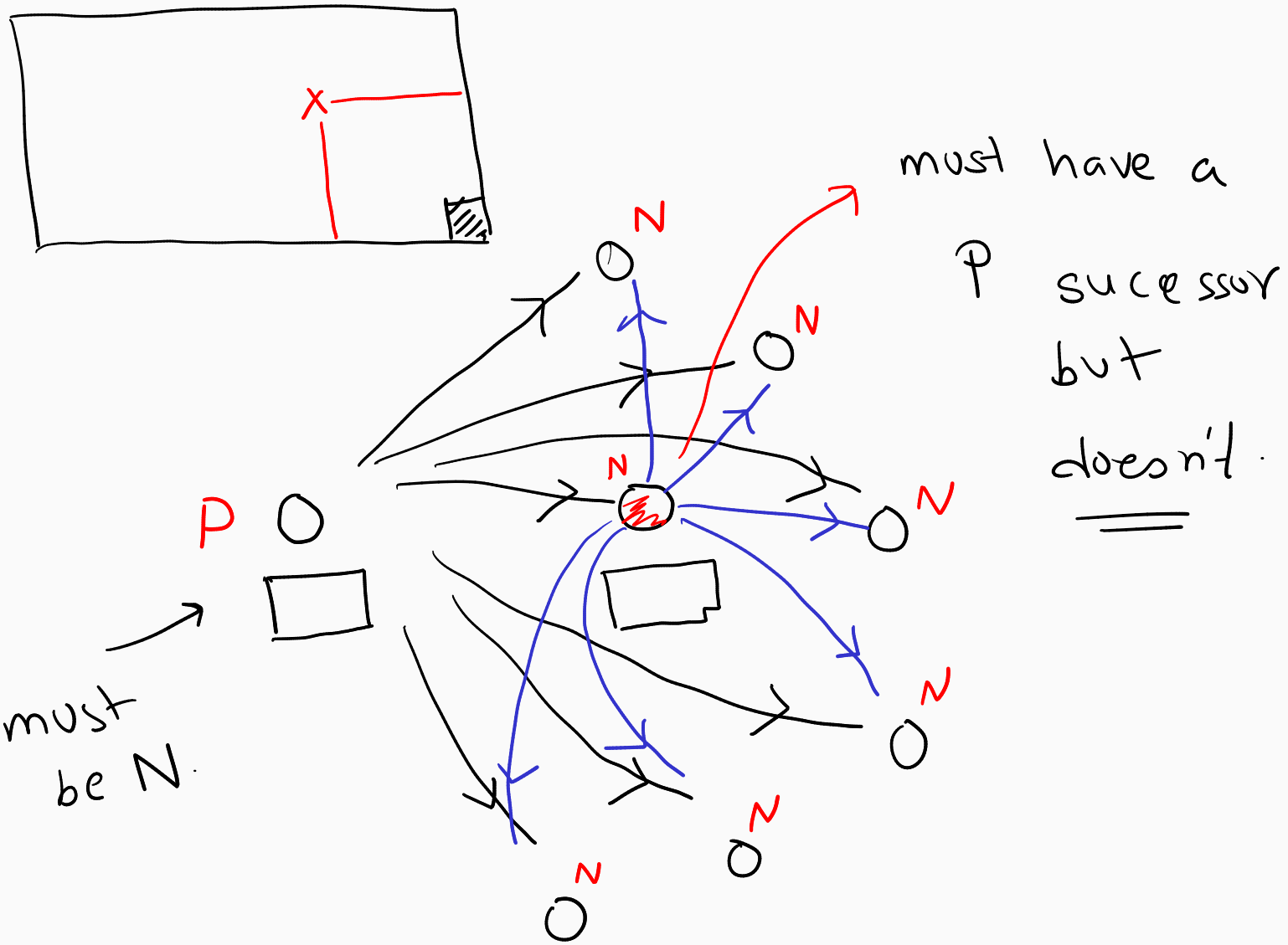
Games, graphs, and machines

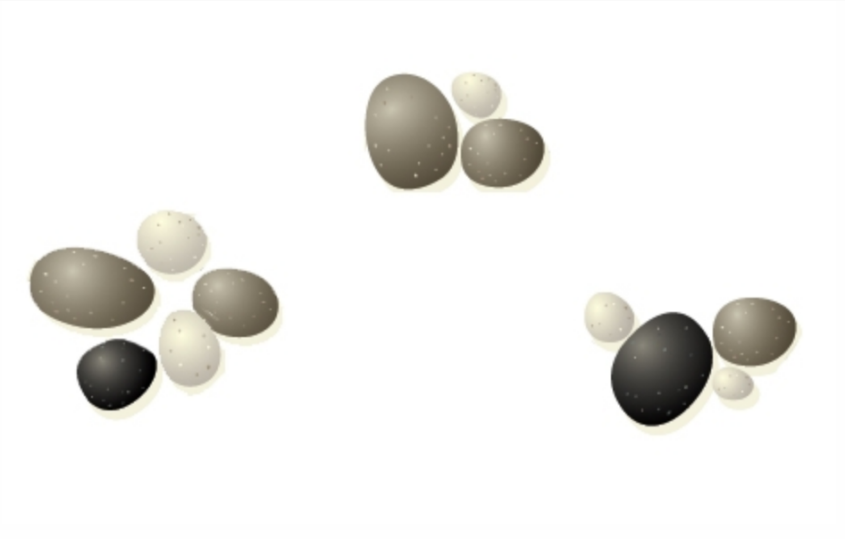
October 9, 2024

Chomp

Strategy stealing



Nim (jian-shizi)



捡石子

Mirroring

Copy player 1

works for n, n nim.

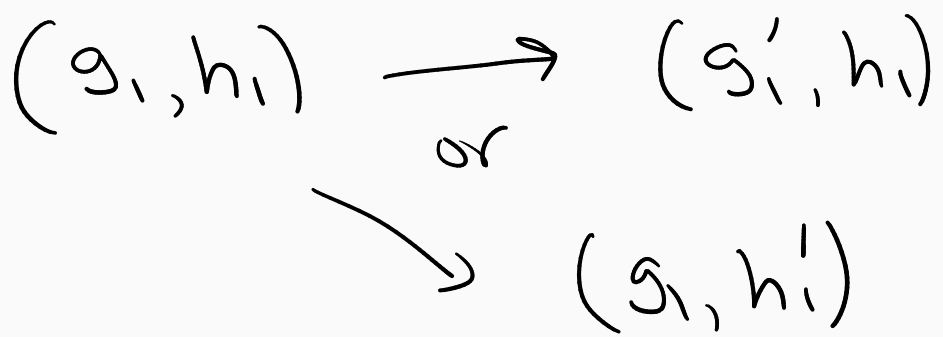
$\{2, 3, 5\}$

$\{n, n\}$ nim is a P- game.

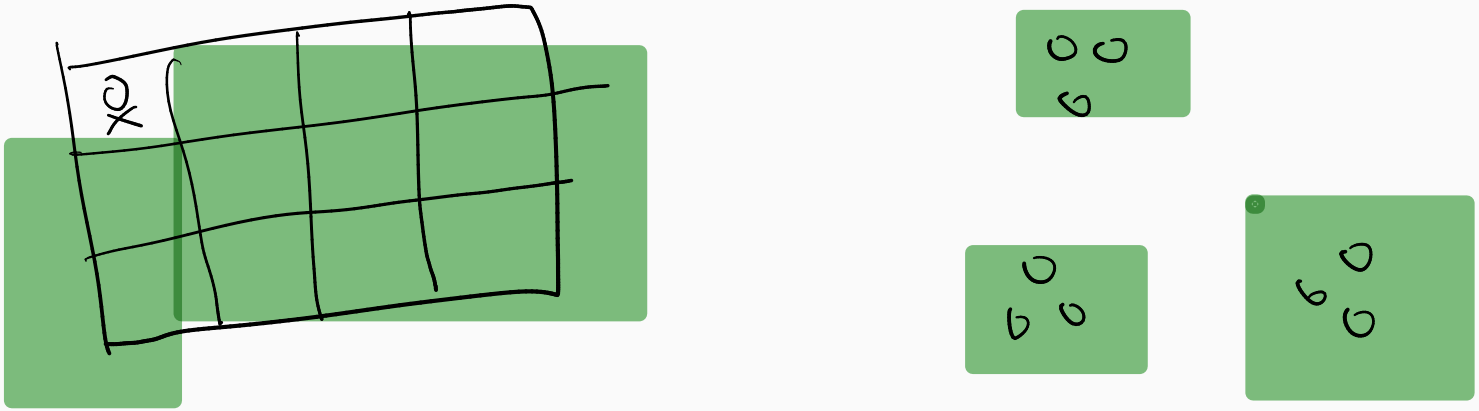
Game sum

- Given G and H, the game $G + H$ is one where a player moves in G or in H, but not both.
- A player unable to make a move loses.

States are States G × States H



Example: Nim + chomp



$$\{a,b,c\}\ \text{nim} = \{a\}\ \text{nim} + \{b\}\ \text{nim} + \{c\}\ \text{nim}.$$

Mirroring: $G + G$ is P

Thm: Let G be any game.

Then $G+G$ is a $\mathcal{P}$-game.

Pf: Second player has a winning Strategy.

Bring the game in $H+H$ form

2,2,4,4 nim

$= \{2,4\}$ nim $+ \{2,4\}$ nim

Hackenbush